Model theory

- Part 1 — 6 weeks
  - 12 lectures
  - + exam
  4 credits
- Part 2 — rest of the time
  - work on a project
  (Around 3–4 pages text, not strict)
  3.5 credits

We get some key-words and have to find some literature

historical + technical overview

(they are graded separately)

The topics are possible continuations of this introductory course

Model theory. theory of mathematical structures. (syed)

— · — from the point of view of FoL (2024)

Born. early 20^{th} century

2 readings

- History of MT (Hodges)
- Review of current topics in MT (Marker)

Abstract Model theory	Applied model theory
- Completeness (Gödel)	- quantifier elimination
~1930 - compactness	- algebraically closed fields
- Lowenheim - Skolem*	- minimality / o-minimality
- Tarski def of semantics	
~1950 - Ultraproducts	
- Morley categoricity* (classification)	
- (Rudsman theorem)	
- Forest theory	

Websites dlberti → teaching - mt 2024

- 4 books (one can choose)
  - Manzano MT (bad notation)
  - MT for beginners (a bit easier) Kossak
  -

4 examples

structure: set equipped with additional data

1) Magmas. $(M, \cdot)$ Associative operation

2) Setoid (M, R) symmetric + reflexive + transitive

3) Non-empty sets

4) Vector spaces $(V, \cdot)$ abelian group, linear action of a field

Universal Algebra

First order languages and FOL structures

- a list of sorts
- a list of relations, functions and symbols
- a list of variables

Structures: Let $\mathbb{L}$ be a FO language

An $\mathbb{L}$-structure is

- for every sort X_i, a set $[X_i]$.
- R^n_i, a relation $[R^n_i] \to X_i^n$
- f
- c

FOL we have $\Rightarrow, \vee, \wedge, \neg$, variables, $=$, $\forall, \exists$

atomic formulas $x_i = x_j$

$R^n_\sigma(x_1, \ldots, x_n)$

formulas atomic

$\neg\varphi$, $\varphi \vee \psi$, $\varphi \wedge \psi$, $\varphi \Rightarrow \psi$, $\exists x \varphi$, $\forall x \varphi$, with φ, ψ formulas

Tarskian semantics

Q Find a language, a structure, and a theory for vector spaces

NEXT: Recall + LS.

$(V, K, +, \cdot, 0, 1)$ $\quad + \quad V \times V \longrightarrow V \quad \cdot \quad V \times K \longrightarrow V$

Refurbed

1) $\forall x, y \in V \quad (x + y = y + x)$

2) $\forall x, y, z \in V \quad (x + (y + z) = (x + y) + z)$

3) $\forall x \in V \quad (x + 0 = x)$

4) $\forall x \in V \ \forall a \in K \quad (a\, x = x \cdot a)$

5) $\forall x \in V \ \forall a, b \in K \quad (a(bx) = (ab)x)$

6) $\forall x \in V \quad (1x = x)$

7) $\forall x \in V \ \exists y \in V \quad (x + y = 0)$

8) $\forall a \in K \ (a = 0 \ \vee \ \exists b \in K \ (ab = 1))$

9)